TECHNICAL MEMORANDUM ASRCN-63-33

TURBULENT FILM CONDENSATION

Jon Lee

May 1963

DIRECTORATE OF MATERIALS AND PROCESSES
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

PROJECT: 7340 ~~3044,~~ TASK: 734008 ~~304403~~

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
May, 1963

3. REPORT TYPE AND DATES COVERED
FINAL January - May 1963

4. TITLE AND SUBTITLE
TURBULENT FILM CONDENSATION

5. FUNDING NUMBERS

6. AUTHOR(S)
JON LEE

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AFB, OH 45433

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AERONAUTICAL SYSTEMS DIVISION
AIR FORCE SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AFB, OH 45433

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
TECHNICAL MEMORANDUM
ASRCN 63-33

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION AVAILABILITY STATEMENT
Approved for public release; Distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
A turbulent film condensation problem was formulated. The physical model and simplifying assumptions are the same as those of W. Nusselt's original investigation, with the exception of including the turbulent transports. The result agrees with the qualitiative results both of R.A. Saban and J. Lee, but does not agree totally with A.E. Duckler.

14. SUBJECT TERMS
LUBRICANTS, FLUIDS, CONDENSATION, LAMINAR FILM

15. NUMBER OF PAGES
36

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

FOREWARD

This report was prepared by the Fluid and Lubricant Materials Branch, Nonmetallic Materials Division, AF Materials Laboratory, Aeronautical Systems Division with Jon Lee as project engineer. The work reported herein was initiated under Project No. 7340, "Nonmetallic and Composite Materials", Task No. 734008, "Power Transmission Heat Transfer Fluids".

This report covers the partial work done during the period from January 1963 to May 1963.

ABSTRACT

A turbulent film condensation problem was formulated. The physical model and simplifying assumptions are the same as those of Nusselt's original investigation, with the exception of including the turbulent transports. The result agrees with the qualitative results both of Seban and Lee, but does not agree totally with Duckler.

R. L. ADAMCZAK, Chief
Fluid & Lubricant Materials Branch
Nonmetallic Materials Division
AF Materials Laboratory

INTRODUCTION

In comparison to the Nusselt's original laminar film condensation theory (10), the higher heat transfer coefficient observed for the fluids of moderate and high Prandtl number and the lower heat transfer coefficient associated with fluids of small Prandtl number lead us to consider the influence of the turbulent transports in condensation. As the length of a condenser plate and/or the rate of the heat transfer (condensation) become large, the thickness of the condensate film is also increased considerably. Consequently, in view of the knowledge of fluid mechanics, for a thicker condensate film it is not unreasonable to conjecture that the transports of momentum and heat energy are likely subject to the turbulent mechanism in addition to what is essentially molecular in nature.

Needless to say, the actual rational approach to this sort of problem presents a serious dilemma due to the fact that we have not yet conquered the deadlock difficulty of the turbulent problem in its own right. In other words, the appearance of the Reynolds (turbulent) shear stress has been the source of distress in the field of turbulent theory; however, the most practical solution has been the phenomenological approach in which the analogy to the random molecular motion plays an essential role with the aid of a judiciously selected eddy diffusivity.

In the past, few attemps have been made in this direction by Seban (12), Rohsenow et al. (11), and Duckler (3). Seban and Rohsenow et al. simply adopted the well-known piece meal logarithmic velocity profiles of the bounded flow for the velocity distribution within the condensate film. With the additional assumption of eddy thermal conductivity being equal to the eddy viscosity, they carried out the heat transfer analysis by a crude analytical method. In view

of the recent work by Lee (6), it is now clear that the use of the logarithmic velocity profile is valid only in the case of a positively large interfacial shear stress. Recently Duckler contemplated a more ambitious program of actually solving for the velocity distribution within the condensate film using the eddy viscosity expressions of Deissler and von Karman. However, his velocity distribution suffers from the traditional physical inconsistencies of being discontinuous at the intersection of two regions and nonsymmetric (for zero interfacial shear stress) **at the center in the velocity gradient.** Moreover, the interfacial shear stress is not satisfied correctly for the non-varnishing values. Furthermore, Duckler seemingly forgot the salient fact that the heat transfer aspect of the film condensation problem is two-dimensional in nature, in the absence of a similarity transformation.

Lee (6) has recently studied the solubility of the equations of motion arising from the Deissler and von Karman eddy viscosities and concluded that a continuous velocity distribution, at least up to the first derivative, can be obtained with an arbitrary interfacial shear stress. This allows us to eliminate the forementioned objections on the velocity distribution within the condensate film with the desired interfacial shear stress being faithfully satisfied.

The purpose of this paper is to apply the correct solution for the velocity distribution to a film condensation problem for which the interfacial shear stress vanishes. Obviously, a problem defying the similarity transformation is more involved than it appears to be, due to the mundane fact that we must maintain the rigorosness of two dimensional formulation.

In retrospect, the general consensus of the experimenters is that a more reliable measurement of the temperature drop across the condensate film can be

observed when the thickness of film becomes large. Certainly, the temperature drop across the condensate film is one of the most sensitive parameters which gravely influence the reliability of the condensation heat transfer data, and any experiments worthy of notice should include such a measurement. However, as the thickness of the condensate film grows, it can no longer be described adequately by the naive Nusselt's laminar film theory, thus presenting the need for the study of a turbulent film condensation problem.

FORMULATION OF PROBLEM

Consider a physical model of film wise condensation on a vertical semi-infinite plate, where the x-axis is parallel and the y-axis is perpendicular to the plate, as shown in Figure 1. If the cold plate is held at an uniform temperature T_W, which is lower than that of the saturated pure vapor T_S, the vapor will form a condensate film. As an initial investigation, let us restrict ourselves to the physical model that was originally adopted by Nusselt. That is, assume the following: (i) consider only the balance of acceleration due to gravity and retardation due to drag on the plate in the equation of motion, (ii) neglect the inertia effect and the interfacial sheer stress at the outer edge of the condensate film, (iii) consider the conduction type mechanism of heat energy transport is dominant in the heat energy equation, and (iv) neglect the convective heat transfer and subcooling in the condensate film. However, we do differ from the original Nusselt's formulation by adding the effect of a turbulent transport in the form of eddy diffusity.

Under the condition for which the above stipulations are faithfully satisfied, we can write the following equations of motion and heat energy within the condensate film. Of course, the usual assumption of the constancy of the physi-

cal properties within the range of temperature involved is evoked:

$$(1)\qquad (\nu + \tilde{\nu}(u,y)) \frac{\partial u}{\partial y} = g(\delta - y)$$

$$(2)\qquad (\kappa + \tilde{\kappa}(u,y)) \frac{\partial T}{\partial y} = \kappa\left(\frac{\partial T}{\partial y}\right)_o$$

where $g = g'(\Delta\rho/\rho)$, and $\tilde{\nu}$ and $\tilde{\kappa}$ denote the eddy viscosity and thermal conductivity whose explicit functional forms are yet to be specified.

In principle, the above equations can be solved for the velocity and temperature distributions in the condensate film which are consistent with following boundary conditions:

$$\left.\begin{array}{l} u = 0 \\ T = T_w \end{array}\right\} \quad \text{at } y = 0 \text{ and for } x \geq 0$$

$$T = T_s \quad \text{at } y = \delta \text{ and for } x \geq 0$$

However, one immediately notices the ubiquitous appearance of δ in the system of equations, which can only be known as a result of analysis. In order to define the a priori undetermined δ, we let $y \to \delta$ in equation 2 to obtain the total heat energy transport in the condensate film, that is:

$$(3)\qquad (\kappa + \tilde{\kappa}(u,y))_\delta \left(\frac{\partial T}{\partial y}\right)_\delta = \kappa\left(\frac{\partial T}{\partial y}\right)_o$$

The growth of condensate film thickness δ, which depends not only on the hydrodynamics but on the phase change (condensation phenomena), can be described by equation 3 if one introduced the following interfacial condition, viz.:

$$(4)\qquad (\kappa + \tilde{\kappa}(u,y))_\delta \left(\frac{\partial T}{\partial y}\right)_\delta = \frac{\lambda}{C_p} \frac{\partial}{\partial x} \int_0^\delta u \, dy$$

Substitution of equation 4 into 3 gives

$$(5) \qquad \frac{\lambda}{C_p} \frac{\partial}{\partial x} \int_0^{\delta} u \, dy \; - \; \kappa \left(\frac{\partial T}{\partial y} \right)_0 = 0$$

Equations 1, 2, and 5 constitute the system of equations we wish to study. Clearly, they are reducible to the original Nusselt's laminar problem, if $\tilde{\nu}$ and $\tilde{\kappa}$ were not present. One can also show (7) that the above set of equations could have been derived rigorously from the standard boundary-layer approximation, if the forementioned simplifying assumptions are properly applied.

For the eddy viscosity, we shall use the following expressions: Deissler's, near the plate (2):

$$(6) \qquad \tilde{\nu}(u,y) = n^2 u y \left[1 - \exp\left(- n^2 u y / \nu \right) \right] \quad , 0 \leq y \leq y^* \text{, and}$$

von Karman's, away from the plate (13):

$$(7) \qquad \tilde{\nu}(u,y) = K^2 \left| \frac{(\partial u / \partial y)^3}{(\partial^2 u / \partial y^2)^2} \right| \qquad , y^* \leq y$$

where n and K are the empirical constants. The region of Deissler and von Karman are separated at y^* which, in fact, is the third disposable constant. For the tube flow, Deissler suggested the value of $y^* = 26\sqrt{\tau_w/\rho}/\nu$ in connection with the asymptotic logarithmic profile;however,a slight different value of y^* should be used here so that the asymptotic logarithmic profile is preserved for a large positive interfacial shear stress (5, 6).

SOME MANIPULATIONS

Let us accept the additional simplifying assumption that the eddy thermal conductivity is a constant multiple of eddy viscosity (for example, unit), then equations 1, 2, and 5 must hold for both regions referred to in equations 6 and 7. Since $\widehat{\nu}(u,y)$ vanishes at the wall we can from equation 1 derive the value of the parameter $y^* = C\,\nu/\sqrt{g\delta}$ (where C is a numerical value), which demarcates the ranges of equations 6 and 7. Along the distance of a plate, i.e., x-axis, the precise value of y^* can not be assigned ab initio, due to its dependency on δ. Nonetheless, only two cases can exist, depending on the value of δ for a certain fixed value of x; that is, $\delta \leq y^*$ and $\delta > y^*$.

Case I -- $\delta \leq y^*$

Physically speaking for this case the transport mechanism in the condensate film is mildly turbulent, i.e., the so-called sublayer and buffer regions. The differential equations for the velocity and temperature distributions can be obtained from equations 1, 2, and 6, viz.:

$$(8) \quad \frac{\partial u_1}{\partial y} = \frac{(g/\nu)(\delta - y)}{1 + (n^2/\nu)\, u_1\, y\, \left[1 - \exp\left(-n^2 u_1 y/\nu\right)\right]}, \quad \delta \leq y^*$$

$$(9) \quad \frac{\partial T}{\partial y} = \frac{(\partial T/\partial y)_0}{1 + (n^2/\nu)\, P_r\, u_1\, y\left[1 - \exp(-n^2 u_1 y/\nu)\right]}, \quad \delta \leq y^*$$

where the velocity distribution in the Deissler's region is denoted by u_1.

With the initial condition of $u_1 = 0$ at $y = 0$, equation 8 presents no

difficulty in the numerical treatment, even though it does not render itself to an analytical solution in a closed form. For the temperature distribution, equation 9 can be integrated at once in y to give:

$$\text{(10)}\qquad \frac{T - T_w}{\Delta T} = \frac{1}{I_1}\int_0^y \frac{dy}{1 + P_r(n^2/\nu)u_1 y\left[1 - \exp(-n^2 u_1 y/\nu)\right]}$$

where $\Delta T = T_s - T_w$ and $I_1 = \int_0^\delta \frac{dy}{1 + P_r(n^2/\nu)u_1 y\left[1 - \exp(-n^2 u_1 y/\nu)\right]}$

To obtain the unique velocity and temperature distributions, we must evoke the physical phenomena of a condensation process. This can be done by applying the Leibnitz rule to equation 5 to yield an alternate form:

$$\text{(11)}\qquad \left[\int_0^\delta \left(\frac{\partial u}{\partial \delta}\right) dy + u(\delta)\right]\frac{d\delta}{dx} = \frac{C_p\,\nu}{P_r\,\lambda}\left(\frac{\partial T}{\partial y}\right)_0$$

The integrand on the left hand side of the above equation can now be obtained from equation 8 with the help of an assumption for the interchangability of the differential with respect to y and δ, that is:

$$\text{(12)}\qquad \frac{\partial V_1}{\partial y} = \frac{(g/\nu)\left\{\left[1 + (n^2/\nu)u_1 y\left(1 - \exp(-n^2 u_1 y/\nu)\right)\right] - (n^2/\nu)(\delta - y)y\left[1 + ((n^2/\nu)u_1 y - 1)\exp(-n^2 u_1 y/\nu)\right]\; V_1\right\}}{\left[1 + (n^2/\nu)u_1 y\left(1 - \exp(-n^2 u_1 y/\nu)\right)\right]^2}$$

where $V_1 = \partial u_1/\partial\delta$.

Therefore, one must now solve equations 8, 10, 11, and 12 simultaneously to obtain u (x, y), T (x, y), and $\delta(x)$.

<u>Case II -- $\delta > y^*$</u>

For this case, we must consider two regions described by the different eddy viscosity formulas; namely, equations 6 and 7, adjoining at $y=y^*$. The

equation of motion valid for $0 \leq y \leq y^*$ is identical to the previous equation 8; however, for $y^* \leq y \leq \delta$, the differential equation must now be obtained from equations 1 and 7. Let us denote the velocity distributions in the Deissler and von Karman regions by u_1 and u_2, respectively; we then have:

$$\text{(13)} \quad \frac{\partial u_1}{\partial y} = \frac{(g/\nu)(\delta - y)}{1 + (n^2/\nu)u_1 y\left[1 - \exp(-n^2 u_1 y/\nu)\right]} \quad , 0 \leq y \leq y^*$$

and

$$\text{(14)} \quad \frac{\partial^2 u_2}{\partial y^2} = \frac{-(K/\sqrt{\nu})(\partial u_2/\partial y)^2}{\sqrt{(g/\nu)(\delta - y) - \partial u_2/\partial y}} \quad , y^* \leq y \leq \delta$$

In obtaining equation 14, the assumptions of $\partial u_2/\partial y \geq 0$ and $\partial^2 u_2/\partial y^2 \leq 0$ were tacitly evoked. Therefore, one must solve equation 13 and 14 to obtain a velocity distribution within the condensate film, which has the desiderata of continuity and symmetry in velocity gradient.

The temperature distribution can also be found by integrating equation 2 with the aid of equations 6 and 7, thus:

$$\text{(15)} \quad \frac{T - T_w}{\Delta T} = \frac{1}{I_2}\int_0^y \frac{dy}{1 + P_r\,\tilde{\nu}(u,y)/\nu}$$

where $I_2 = \int_0^\delta \frac{dy}{1 + P_r\,\tilde{\nu}(u,y)/\nu}$, for which $\tilde{\nu}(u, y)$ takes the different forms given by equations 6 and 7 depending on the value of y.

The corresponding integrand expressions, appearing in equation 11, must be derived from equations 13 and 14, respectively:

(16)

$$\frac{\partial V_1}{\partial y} = \frac{(g/\nu)\left\{\left[1 + (n^2/\nu)u_1 y\,(\,1 - \exp(-n^2 u_1 y/\nu\,))\right] - (n^2/\nu)(\delta - y\,)\,y\,\left[1 + ((n^2/\nu\,)u_1 y - 1)\,\exp(-n^2 u_1 y/\nu\,)\right]\;V_1\right\}}{\left[1 + (n^2/\nu\,)u_1 y\,(\,1 - \exp(-n^2 u_1 y/\nu\,))\right]^2}\quad, 0 \leq y \leq y^*$$

and

(17)

$$\frac{\partial^2 V_2}{\partial y^2} = \frac{-\frac{K}{\sqrt{\nu}}\left\{2\left[\frac{g}{\nu}(\delta - y) - \frac{\partial u_2}{\partial y}\right]\left(\frac{\partial u_2}{\partial y}\right)\left(\frac{\partial V_2}{\partial y}\right) - \frac{1}{2}\left(\frac{\partial u_2}{\partial y}\right)^2\left[\frac{g}{\nu} - \frac{\partial V_2}{\partial y}\right]\right\}}{\left[\frac{g}{\nu}(\,\delta - y\,) - \frac{\partial u_2}{\partial y}\right]^{3/2}}\quad, y^* \leq y \leq \delta$$

where $V_1 = \partial u_1/\partial\delta$ and $V_2 = \partial u_2/\partial\delta$. As before, the expression for $\delta(x)$ can be derived from equation 11 with the properly defined functions connecting continuously at $y = y^*$.

METHOD OF SOLUTION

All the equations developed in the previous section are too cumbersome to be treated analytically; therefore, one must take a recourse to the numerical method. For either case, the temperature function appearing in equation 11 can be eliminated by substituting either equation 10 or 15 to yield:

(18)

$$\left[\int_0^{\delta} V_{1,2}\,dy + u_{1,2}(\delta)\right] I_{1,2}\,\frac{d\delta}{dx} = \frac{(C_p\,\Delta T/\lambda)\,\nu}{P_r}$$

where the subscript 1 or 2 corresponds to cases I or II, respectively.

Thus, in essence, the mathematical statement of the problem is to find a solution of the above first order differential equation, i.e., $d\delta/dx = F(\delta)$, with the initial condition of $\delta = 0$ at $x = 0$. As can easily been seen, the subtle complication arises from the fact that $F(\delta)$ must be found rigorously by solving the equation of motion.

From the numerical standpoint, the following scheme was adopted for conveniency: (i) Integrate equation 18 to obtain.

$$(19) \quad \int_0^{\delta} \left[\int_0^{\delta} v_{1,2}\, dy + u_{1,2}(\delta) \right] I_{1,2}\, d\delta = \frac{(C_p \Delta T/\lambda)\, \nu\, x}{P_r}$$

(ii) For a given value of δ, the integrand of the above can be evaluated uniquely from either equations 8 and 12, or equations 13, 14, 16, and 17, depending on the value of y^*. Having evaluated the integrand for the preassigned, arbitrarily small, interval of δ, the left hand side of equation 19 can then be computed numerically. (iii) For each δ, a corresponding value of x can be found from the equality, provided the value of parameters, $(C_p \Delta T/\lambda)\nu/P_r$, is specified. (iv) However, in reality we are seeking the value of δ for a regular interval of x, instead: This can easily be done by the use of an interpolation for the fixed x interval with sufficient accuracy. This sort of numerical technique seems to be quite amenable from the fact that the differential equation is of separable type.

Let us now briefly return to the discussion of evaluating the integrand of equation 19 from the equations of motion. For case I, no difficulty arises on account of the regular behavior of equation 8 and 12 within the entire range, $0 \leq y \leq \delta < y^*$. For case II, the velocity distribution must be described

by equations 13 and 14, and the $V_{1,2}$ by equations 16 and 17.

For the velocity distribution within $0 \leq y \leq y^*$, equation 13 can be solved numerically with the initial condition of $u_1 = 0$ at $y = 0$. However, within the range of $y^* \leq y \leq \delta$, equation 14 is expressed into a corresponding system of first order differential equations as:

$$\text{(20)} \qquad \frac{\partial u_2}{\partial y} = P \qquad , \quad y^* \leq y \leq \delta$$

$$\text{(21)} \qquad \frac{\partial P}{\partial y} = \frac{-(K/\sqrt{\nu})\, P^2}{\sqrt{(g/\nu)(\delta - y) - P}}$$

Usually the system of two first order differential equations can satisfy only two initial (or boundary) conditions. Fortunately, due to the singularity of equation 21 at $y = \delta$ being a nodal point, it has been shown that the following three necessary conditions can be fulfilled (5):

(i)	$u_2 = u_1$	at $y = y^*$	(continuity)
(ii)	$P = \partial u_1/\partial y$	at $y = y^*$	(smoothness)
(iii)	$P = 0$	at $y = \delta$	(symmetry)

Thus, the exact solution is continuous up to the first derivative at $y = y^*$, and has a vanishing gradient at $y = \delta$. Therefore, the traditional physical inconsistencies on the velocity distribution are now removed in toto.

Similarly, the $V_{1,2}$ must be evaluated by equations 16 and 17. Again for $0 \leq y \leq y^*$, the solution for equation 16 offers no mathematical handicap.

On the other hand for $y^* \leq y \leq \delta$, equation 17 can be expressed as a corresponding system of first order equations, as:

$$(22) \qquad \frac{\partial V_2}{\partial y} = Q$$

$$, \; y^* = y = \delta$$

$$(23) \qquad \frac{\partial Q}{\partial y} = \frac{-(K/\sqrt{\nu})\left\{2\left[(g/\nu)(\delta - y) - P\right]PQ - P^2((g/\nu) - Q)/2\right\}}{\left[(g/\nu)(\delta - y) - P\right]^{3/2}}$$

The singular behavior of equation 21 is preserved in equation 23 also, thus allowing us to satisfy the following three boundary conditons:

(i)	$V_2 = V_1$	at $y = y^*$
(ii)	$Q = \partial V_1 / \partial y$	at $y = y^*$
(iii)	$Q = g/\nu$	at $y = \delta$

As shown in the Appendix, equation 23 has the point $y = \delta$ as a singular nodal point so that the family of solutions, Q, converges to g/ν as $y \to \delta$.

Having obtained the solution for $\delta(x)$ and u (x, y), the temperature distribution T (x, y) can be computed from equation 10 or 15. Therefore, the solution of the simultaneous equations describing the turbulent film condensation is completed.

HEAT TRANSFER RESULTS

Even though the velocity and temperature distributions within the condensate film and the corresponding film thickness are the fundamental functions describing the condensation phenomena, it has been the accepted practice in the past to express it in terms of the auxiliary parameters, such as, Nusselt and Reynolds numbers. The obvious reason is that the actual measurement of the steep velocity and temperature distributions within an extremely thin condensate film is hopelessly unyielding; but the overall description of a condensation process can be made via the heat transfer coefficient and flow rate, which render themselves to experimental confirmation.

The usual definition of condensation heat transfer coefficient is obtained from the following relationship:

(24) $$H = h \, \Delta T = k \left(\partial T / \partial y \right)_o$$

Thus from equations 10 or 15

(25) $$h = \frac{k}{\Delta T} \left(\frac{\partial T}{\partial y} \right)_o = \frac{k}{I_{1,2}}$$

This can be compared with the laminar film heat transfer coefficient:

(26) $$h_o = k / \delta_o$$

where the subscript o refers to Nusselt's laminar problem and

$$\delta_o = \left[4(C_p \Delta T/\lambda)\ x\ /P_r\ (g/\nu^2) \right]^{1/4} \tag{27}$$

Therefore, if the Nusselt number is defined as $Nu = hL/k$, the ratio of actual Nusselt number to that of Nusselt's laminar case becomes, from equations 25 and 26:

$$Nu\ /\ Nu_o = \delta_o\ /\ I_{1,2} \tag{28}$$

From the velocity distribution, the corresponding ratio of flow rates, which is defined as $\Gamma = \rho \int_o^\delta u\ dy$, can be expressed as:

$$\Gamma/\Gamma_o = \int_o^\delta u\ dy \Big/ (g/\nu)\ \delta_o^3\ /3 \tag{29}$$

Even though the expressions like equations 28 and 29 are quite instrumental for the case of similarity (see Ref. 1, 4, 7), they are not recommended in our problem because the ratios vary along the x-axis (plate length). Consequently, the value of x must be treated as another parameter, which would then defy the generality of the correlation scheme.

To circumvent this inconvenience, let us introduce the following parameters, Reynolds number and the average heat transfer coefficient, defined as:

$$Re_L = 4\Gamma/\mu \Big|_{x=L} = (4/\nu) \int_o^\delta u\ dy \Big|_{x=L} \tag{30}$$

and

$$(31) \qquad (\nu^2/g)^{1/3} h_{av}/k = (\nu^2/g)^{1/3} \, (1/L)\int_0^L dx/\, I_{1,2}$$

With a parabolic velocity distribution and linear temperature distribution of Nusselt's laminar problem, the following relationship exists:

$$(32) \qquad (\nu^2/g)^{1/3} \, h_{av}/k = 1.47 \, Re_L^{-1/3}$$

DISCUSSION

For a wide range of values for P_r and $C_p \Delta T/\lambda$, u (x, y), T (x, y), and $\delta(x)$ were computed numerically using L = 15 cm and $\nu = 0.005$ cm^2/sec (a median value for water and liquid metals). In view of the previous investigation (5), the appropriate numerical values for n = 0.124, K = 0.4, and $y^* = 23 \, \nu / \sqrt{g\delta}$ were adopted here such that the asymptotic logarithmic velocity profile is preserved for a positive, huge interfacial shear stress.

The actual turbulent film thickness, $\delta(x)$, was plotted in Figure 2 for two widely differing values of P_r; and the comparison was made with the corresponding laminar case $\delta o(x)$. For $P_r = 1.0$, the turbulent film thickness deviates from the laminar case insignificantly because of $\delta < y^*$; however, for $P_r = 0.001$, the condition of $\delta > y^*$ prevails for most of x. Therefore, the turbulent film thickness is much larger than $\delta o(x)$ due to the turbulent mixing. Nonetheless, the presence of turbulent transports contributes to the general increase of the film thickness for all cases. Qualitative information can be deduced by simply letting $\delta \sim Cx^{1/4}$; then we have $y^* \sim C^{-1/2} \, x^{-1/8}$ (where

C has a numerical value). For $P_r = 1.0$, C is small and y^* lies above the $\delta(x)$ as shown in Figure 3 (a). On the other hand, for $P_r = 0.001$ a larger value of $\delta(x)$ (or C) results in a small y^* which then lies below $\delta(x)$, as in Figure 3(b). Crudely speaking, y^* separates the condensate film into the so-called sublayer-buffer region and turbulent core region. In the former region, the influence of the Deissler's eddy viscosity is rather unnoticeable; however, in the latter region, the turbulent mixing caused by the von Karman's eddy viscosity increases the film thickness enormously. This will in turn decrease the value of y^* in the reciprocal relationship. Therefore, it is more appropriate to interprete the two cases of Figure 2 as the difference in the controlling regions, $\delta < y^*$ or $\delta > y^*$, rather than in the P_r values.

Some typical velocity and temperature distributions for $C_p \Delta T/\lambda = 0.04$ and x = 15 cm are presented in Figures 4 and 5. In Figure 4 for $P_r = 10$ and 1, the milder influence of turbulent transports is manifested by the nearly parabolic profile; but as P_r decreases, the inclusion of the von Karman region gives rise to a flatter velocity profile due to the violent turbulent mixing. In contrast to the previous attempts (3, 11, 12)*, one must note that the present analysis yields a velocity distribution whose gradient not only is continuous across the film thickness but also vanishes at the outer edge of the film.

The corresponding temperature distributions are shown in Figure 5, for

---

* We shall not repeat the discussion on the velocity profile for a turbulent film flow because it has already been presented elsewhere (6), in which the works of ref. (3, 11, 12) are carefully scrutinized.

which the general trend with respect to P_r follows what has already been expounded qualitatively in the previous works (8, 12). That is, as the value of P_r becomes small, the magnitude of molecular thermal conductivity is dominant and moreover, far from being overshadowed by the eddy thermal conductivity. Thus, the linearity of the temperature profile is preserved in the limit. For $P_r = 10$ and 1, the temperature profile is convex over the diagonal (linear profile); however, as P_r decreases, the temperature distribution twists around the diagonal and finally coincides closely with the linear profile for $P_r = 0.001$.

For the reason mentioned in the previous section, we shall not present the condensation result in terms of Nu/Nu_o and Γ/Γ_o, as given by equations 28 and 29. However, the correlation of the average heat transfer coefficient and Reynolds number has been adopted for our work, and the Figure 6 contains the result of computation for a wide variation of P_r. For $P_r \geqslant 0.1$, the average heat transfer coefficient is larger than that of the laminar case, but a decrease is observed for smaller values of P_r. It seems apparent that the average heat transfer coefficient approaches the asymptote as P_r becomes vanishingly small. The result of Figure 6 generally agrees with the qualitative study of Seban (12), and the discrepancy is mainly attributed to the approximate use of asymptotic logarithmic velocity profile and the subsequent analytical treatment.

On the other hand, there is a legitimate ground for expecting to see a rather closer agreement with Duckler's no interfacial shear stress case, because we not only have the same physical model but also use identical eddy viscosity expressions. At most, however, there can be a minor degree of dis-

agreement arising from the difference in computing the velocity distribution; namely, that of exact and approximate methods as discussed in ref. (6). Figure 7, containing the results of no interfacial shear stress, was taken from Duckler's paper, in which the curve for $P_r = 0.05$ was included. In conformity with the results of Seban and this work, the average heat transfer coefficient is greater than the laminar case for $P_r \geqslant 1$. However, as P_r becomes smaller, the decrease of the average heat transfer coefficient is exceedingly drastic so that there appears to be no limit as P_r approaches zero. A definite comparison was made by carrying out the computation for the set of parameters used by Duckler. The result of computation is shown for comparison in Figure 7 with $n = 0.1$, $K = 0.4$, and $y^* = 20\, \nu/\sqrt{g\delta}$. Clearly, for $P_r \geqslant 1$, the divergence is not so critical; however, for the smaller values of P_r, Duckler's curves not only have no similarity to the exact result but also decrease without limit. Since Duckler's $P_r = 0.05$ curve passes through the aggregate of liquid metals data (9), the unexplainable lowering of the average heat transfer coefficient for liquid metals condensation was once considered as being caused by the unaccounted turbulent effect. Unfortunately, in light of the above discussion, such a conclusion does not seem readily acceptable. The dramatic discrepancy is obviously caused by the aberrant analysis of Duckler, in which the two dimensional feature of the condensation problem was not taken into account rigorously and the neglect of the molecular thermal conductivity in comparison to the eddy thermal conductivity was not justified. Indeed, the latter is the prime contributor to the veritable fact that the Duckler's heat transfer coefficient is much smaller than the exact result, because the molecular thermal conductivity is far superior to the eddy thermal conductivity as P_r decreases.

CONCLUSIONS

With hope of clarifying the notorious source of inconsistently small heat transfer coefficients observed in liquid metals condensation, the original Nusselt's problem was modified to include the effect of turbulent transports.

The following conclusions can be deduced:

i) The thickness of turbulent condensate film is always larger than that of the laminar case;

ii) The eddy viscosity tends to flatten the velocity distribution in the turbulent core region, yet the parabolic profile is observed for the sublayer-buffer region;

iii) For a large P_r, the temperature distribution becomes convex over the diagonal, which indicates the violent mixing within the condensate film; however, essentially linear distribution is observed for a smaller P_r, in which case the molecular thermal conductivity is still dominant;

iv) The heat transfer result of the present investigation agrees with the qualitative studies of both Seban and Lee, but a considerable amount of disagreement is observed with Duckler's work; and

v) The reported liquid metals condensation data of Misra and Bonilla is still lower than the present theorgy, in contrast to Duckler's claim.

Appendix

SINGULAR BEHAVIOR OF EQUATIONS 19 AND 21

With the choice of the dimensionless coordinates $\eta = y/\delta$, $\psi = P/(g\delta/\nu)$, and $\phi = Q'(g/\nu)$, equations 19 and 21 can be reduced to the following dimensionless forms:

$$\text{(A-1)} \qquad \frac{d\psi}{d\eta} = \frac{-C\,\psi^2}{\sqrt{(1-\eta)-\psi}}$$

$$\text{(A-2)} \qquad \frac{d\phi}{d\eta} = \frac{-C\left\{2\left[(1-\eta)-\psi\right]\psi\phi - \psi^2(1-\phi)/2\right\}}{\left[(1-\eta)-\psi\right]^{3/2}}$$

where $C = K\sqrt{g\,\delta^3}\,/\nu$.

The nodal singular behavior of equation A-1 was investigated previously by a graphical method (5). We shall here develop an analytical expression for ψ at $\eta \cong 1$, which can, in turn, be useful for the study of equation A-2 at the singular point. The singular point can be brought into the origin by letting $\xi = 1 - \eta$, in equations A-1 and A-2:

$$\text{(A-3)} \qquad \frac{d\psi}{d\xi} = \frac{C\,\psi^2}{\sqrt{\xi-\psi}}$$

$$\text{(A-4)} \qquad \frac{d\phi}{d\xi} = \frac{C\left\{2(\xi-\psi)\psi\phi - \psi^2(1-\phi)/2\right\}}{(\xi-\psi)^{3/2}}$$

By introducting a new variable $\omega = \xi - \psi$, equation A-3 can be transformed into a more convenient form:

$$\text{(A-5)} \qquad \frac{d\omega}{d\psi} = \frac{\sqrt{\omega}}{c\,\psi^2} - 1 \qquad (\omega = 0 \text{ at } \psi = 0)$$

For a small value of ψ, a series solution of the above equation can be developed as $\omega = C^2\psi^4 + \cdots$, from which a solution of equation A-3 in an implicit form becomes:

$$\text{(A-6)} \qquad \xi = \psi + c^2\psi^4 + \ldots$$

As before, the nodal singular behavior at the origin, i.e., $\psi = 0$ at $\xi = 0$, is repeatedly established.

Since near the origin, the solution A-6 permits us to approximate $(\xi - \psi) \cong C^2\psi^4$ and $\psi \simeq \xi$, equation A-4 can now be simplified as,

$$\text{(A-7)} \qquad \frac{d\phi}{d\xi} = \frac{2\,c^2\xi^5\phi - \xi^2(1-\phi)/2}{c^2\xi^6}$$

or,

$$\text{(A-8)} \qquad \frac{d\phi}{d\xi} - 2\left[\frac{1}{\xi} + \frac{1}{4\,c^2\xi^4}\right]\phi + \frac{1}{2\,c^2\xi^4} = 0$$

For a small value of ξ, $1/\xi \ll 1/4c^2\xi^4$, then equation A-8 can be simplified further:

$$\text{(A-9)} \qquad \frac{d\phi}{d\xi} = \frac{1}{2\,c^2\xi^4}(\phi - 1)$$

With the introduction of the transformation $\hat{\phi} = \phi - 1$, equation A-9 reduces to:

$$(A\text{-}10) \qquad \frac{d\hat{\phi}}{d\xi} = \frac{1}{2\,C^2\,\xi^4}\,\hat{\phi}$$

The solution of the above equation is:

$$(A\text{-}11) \qquad \hat{\phi} = \hat{\phi}_o \exp\left(- 1 / 6\,C^2\,\xi^3 \right)$$

where $\hat{\phi}_o$ represents the integration constant.

It can be deduced from the solution, A-11, that $\hat{\phi} = 0$ as $\xi \to 0$, which implies $\phi = 1$ at the origin. This concludes the argument that a solution of equation A-2 converges to $\phi = 1$ at $\eta = 1$, or $Q = g/\nu$ at $y = \delta$, in unison. The singular nodal behavior of equation 21 in that a family of solutions converges to $Q = g/\nu$ at $y = \delta$ can be established, perhaps by a an isocline study as was done for the case of equation 19 (5).

Notations

C_p	specific heat at constant pressure
F( )	function
g	$g' (\Delta \rho / \rho)$
g'	acceleration due to gravity
H	amount of heat transferred
h	condensation heat transfer coefficient
I	integral (see equations 10 and 15)
K	von Karman's constant
k	absolute thermal conductivity
L	plate length
n	Deissler's constant
P	$\partial u / \partial y$
P_r	Prandtl number (ν / κ)
Q	$\partial V / \partial y$
Re_L	Reynolds number at x = L
T	temperature
ΔT	$T_s - T_w$
u	velocity component in x-axis
V	$\partial u / \partial \delta$
x, y	coordinate system
y*	separation of Deissler and von Karman regions

Greeks

Γ	flow rate
δ	condensate film thickness
η	y/δ
κ	thermal conductivity $(k/C_p\rho)$
λ	laten heat vaporization
μ	absolute viscosity
ν	kinematic viscosity
ξ	$1 - \eta$
ρ	density
$\Delta\rho$	density of liquid - density of vapor
τ	shear stress
ϕ	$Q/(g/\nu)$
$\hat{\phi}$	$\phi - 1$
ψ	$P/(g\delta/\nu)$
ω	$\xi - \psi$

subscripts

av	average over x
o	Nusselt's laminar case
s	saturation
w	wall

overscore

$\sim$	turbulent property

References

1. Chen, M. M., ASME, Jour. Heat Trans., 83, 48 (1961)

2. Deissler, R. G., NACA TN 3145 (1954)

3. Dukler, A. E., Chem. Engr. Prog. Symp. Series, No. 30, 56, 1 (1960), and the appendix (ADI - 6058)

4. Koh, J. C. Y., Sparrow, E. M., and Hartnett, J. P., Int. Jour. Heat and Mass Transfer, 2, 69 (1961)

5. Lee, J., submitted to AIAA Jour. (Tech. Memo, ASRCN 63-25, WPAFB)

6. Lee, J., submitted to Trans ASME, Jour. Appl. Mech. (Tech. Memo, ASRCN 63-26, WPAFB)

7. Lee J., submitted to Trans ASME, Jour. Heat Trans. (Tech. Rept ASD_TDR_62-1048, part II, WPAFB)

8. Lee, J., Tech. Rept, ASD_TDR_62-1048, part I, WPAFB

9. Misra, B. and Bonilla, C. F., Chem. Engr. Prog. Symp. Series, No. 18, 52, 7 (1956)

10. Nusselt, W., Zeit. Ver. Deutsch Ingr. 60, 541 and 569 (1916)

11. Rohsenow, W. M., Webber, J. H., and Ling, A. T., Trans ASME, 78, 1637 (1956)

12. Seban, R. A., Trans ASME, 76, 299 (1954)

13. von Karman, Th., Trans ASME, 61, 705 (1939)

Acknowledgments

The author wishes to express his sincere appreciation to Mr. Robert Benzing for his interest and encourgement for this work. Many thanks to Mr. Carroll Fetters who gave untiring assistance in debugging the computer programming, and Mr. Karl Mecklenburg for his help in preparing the manuscript. Finally, thanks are due to Mr. Joe Miller for his superb draftmanship.

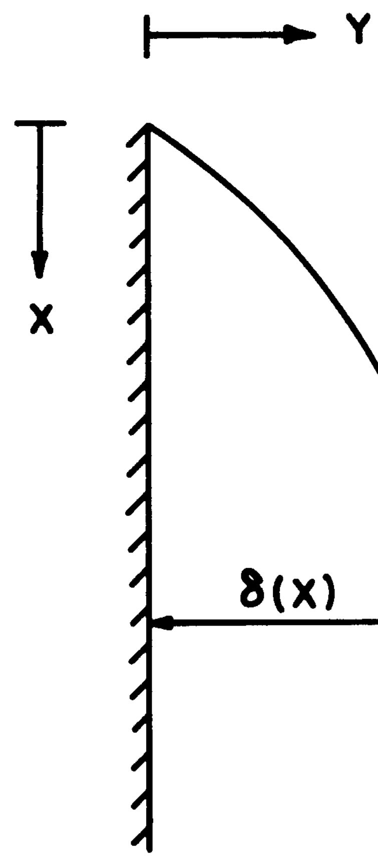

FIGURE 1 - MODEL AND COORDINATES

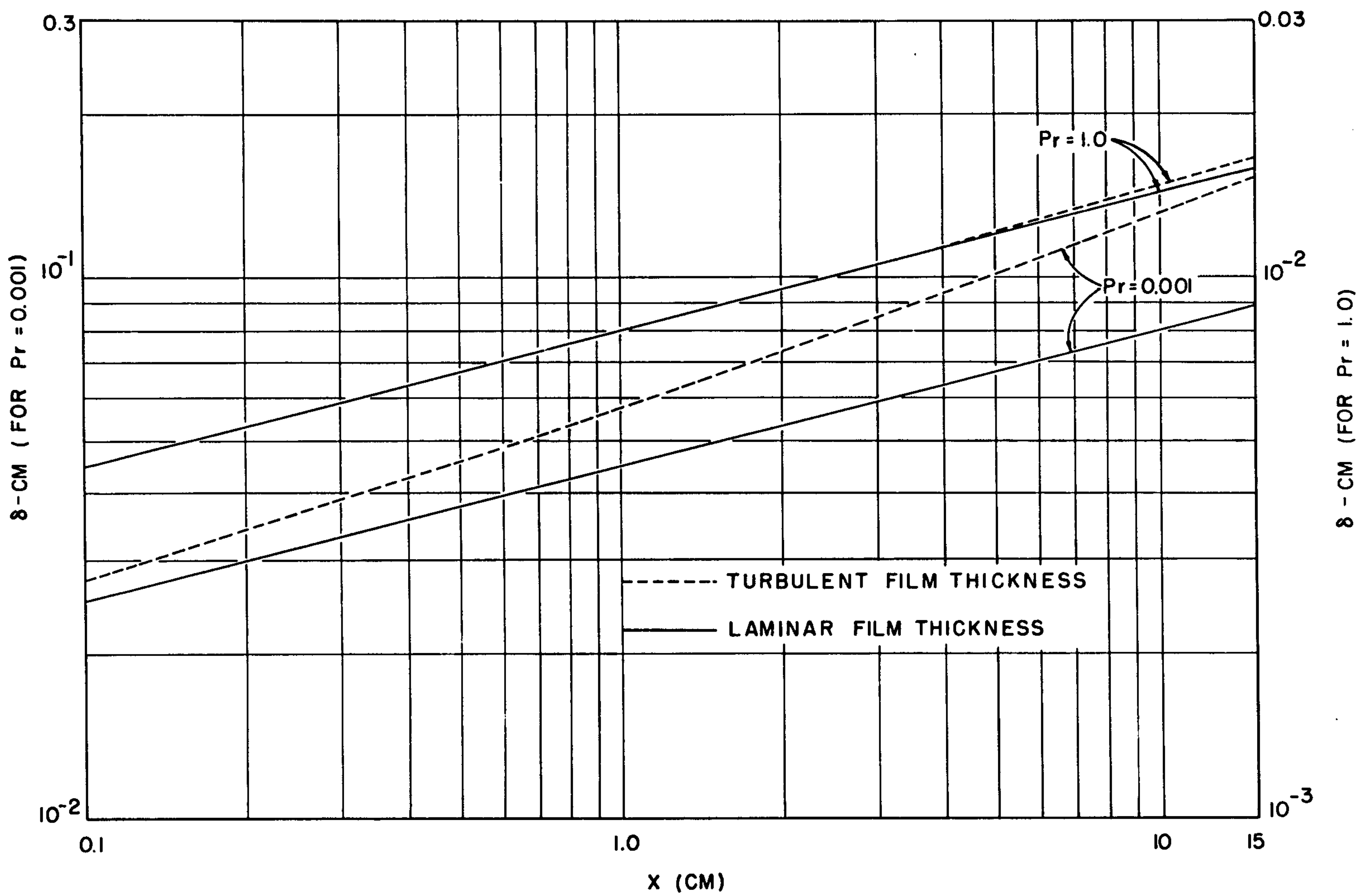

FIGURE 2 - FILM THICKNESS $\left(\frac{C_p \Delta T}{\lambda} = 0.04\right)$

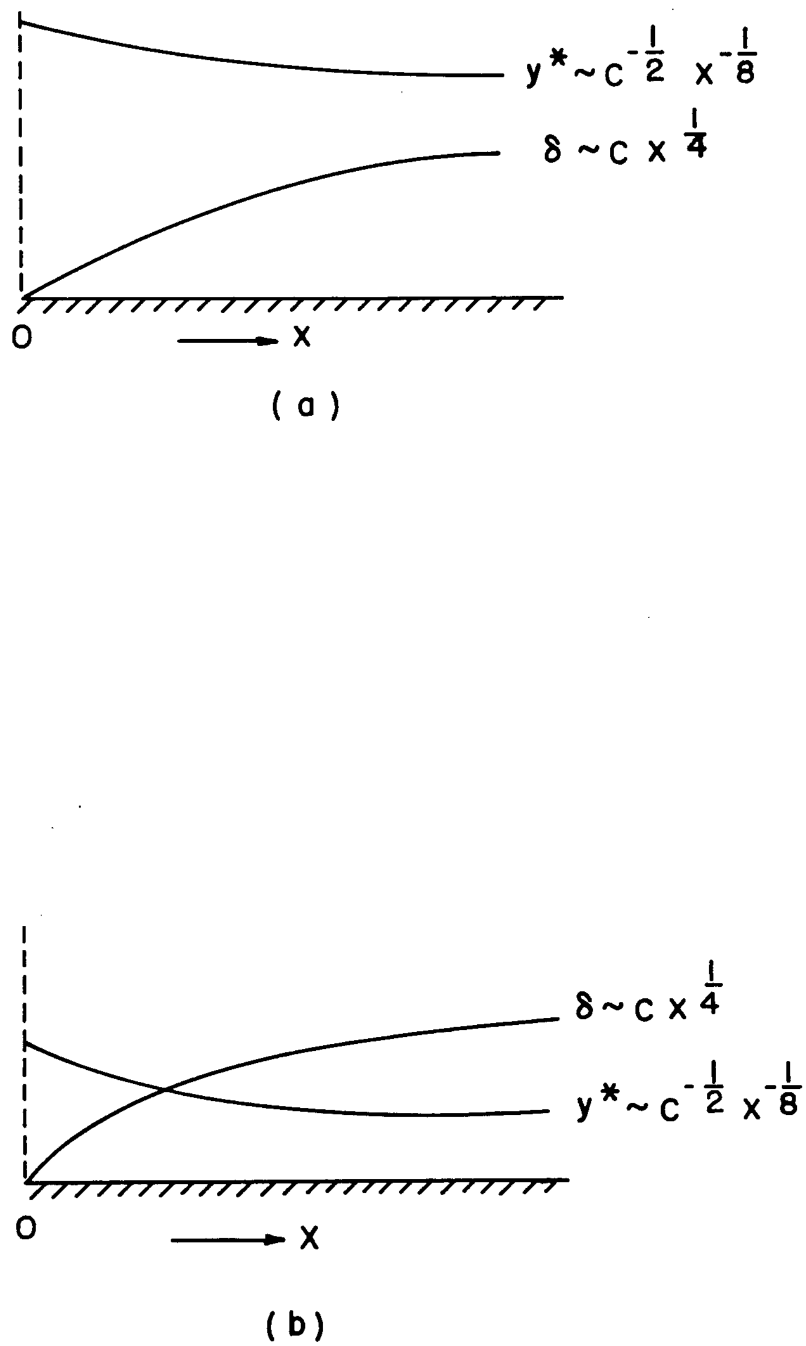

FIGURE 3 - SEPARATION OF CONDENSATE FILM

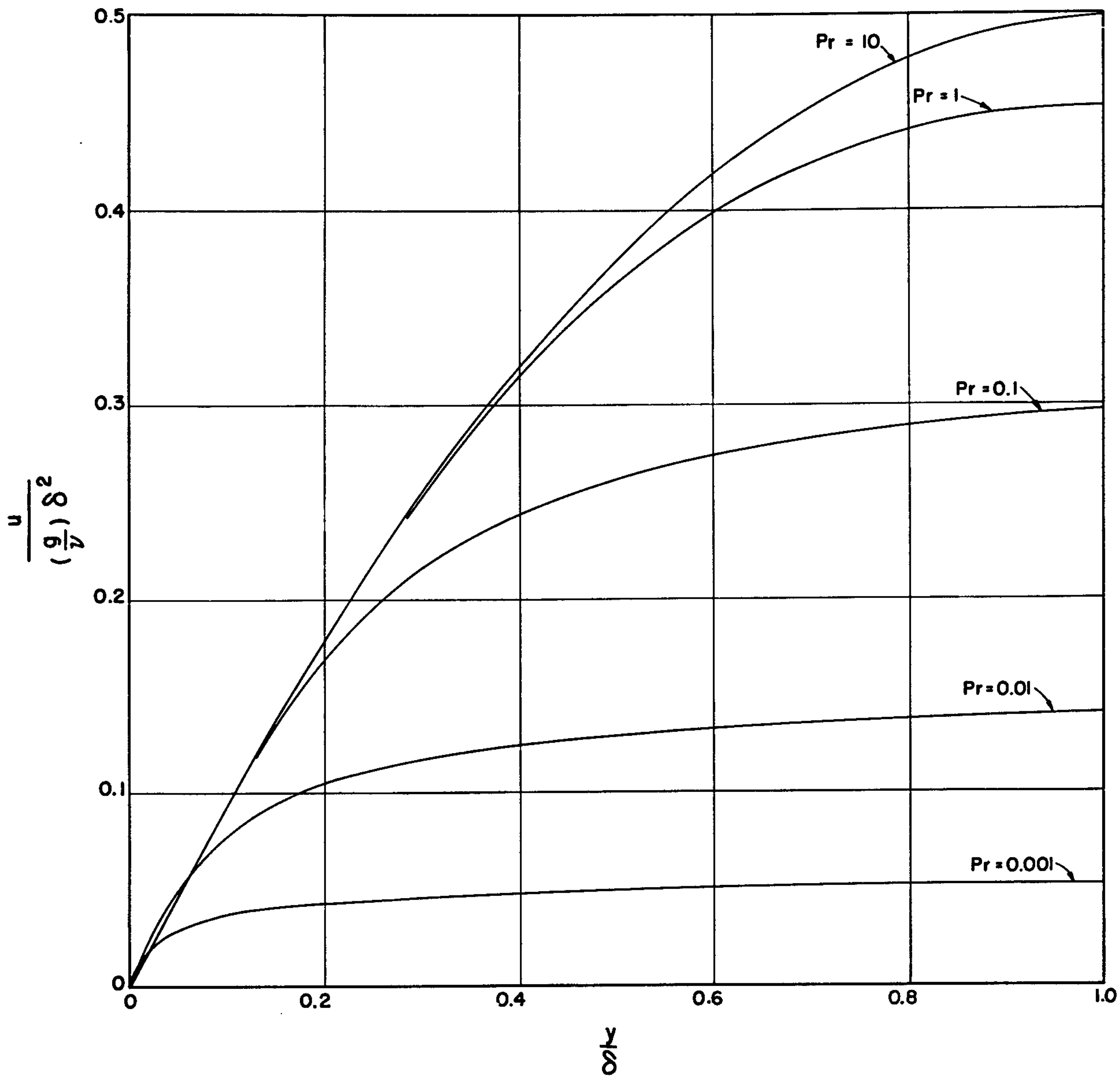

FIGURE 4- VELOCITY DISTRIBUTION $\left(\frac{C_p \Delta T}{\lambda} = 0.04 \text{ \& } X = 15\text{CM}\right)$

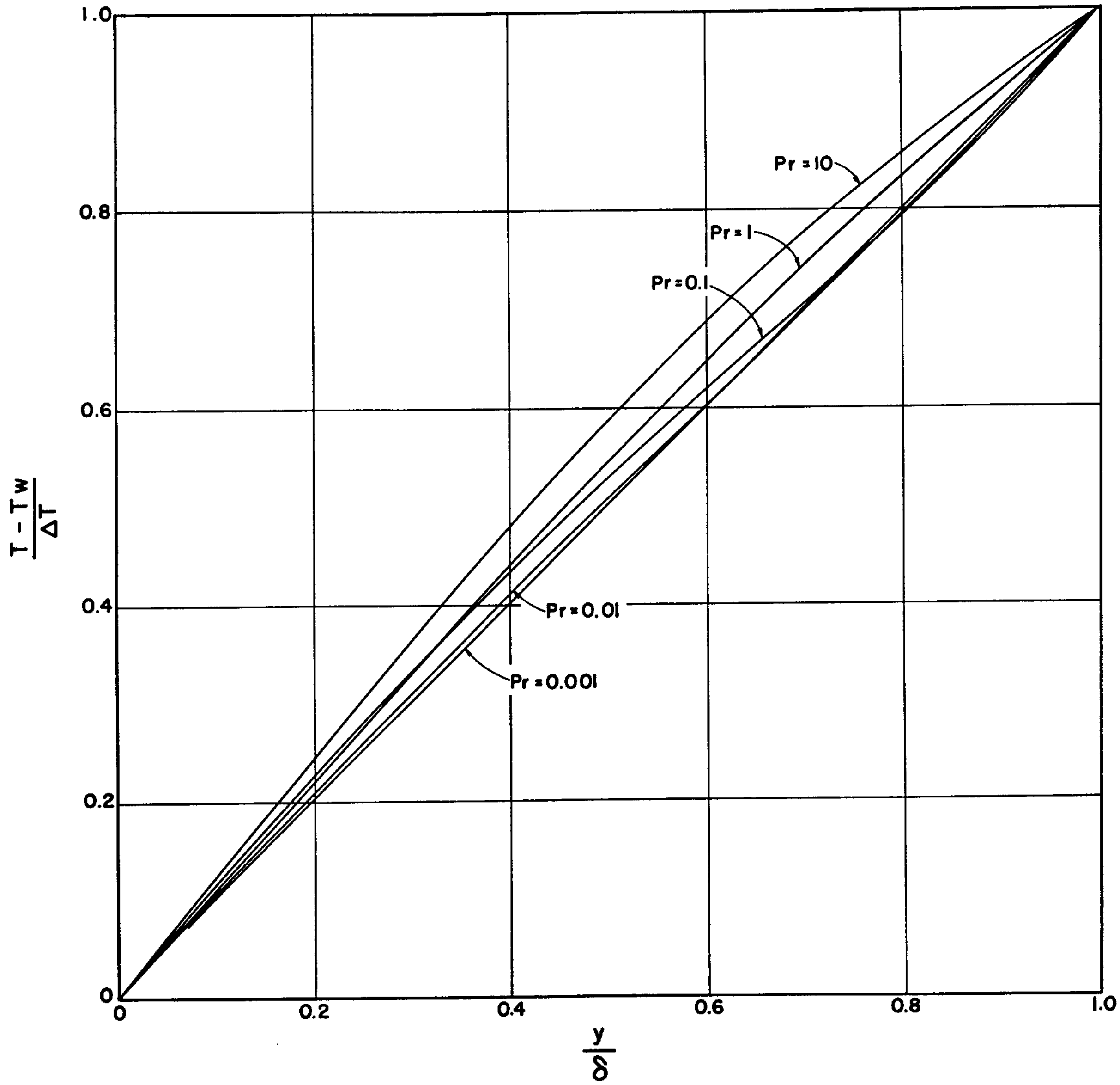

FIGURE 5- TEMPERATURE DISTRIBUTION $\left(\frac{C_p \Delta T}{\lambda} = 0.04\right.$ & $\left. X = 15 \text{ CM}\right)$

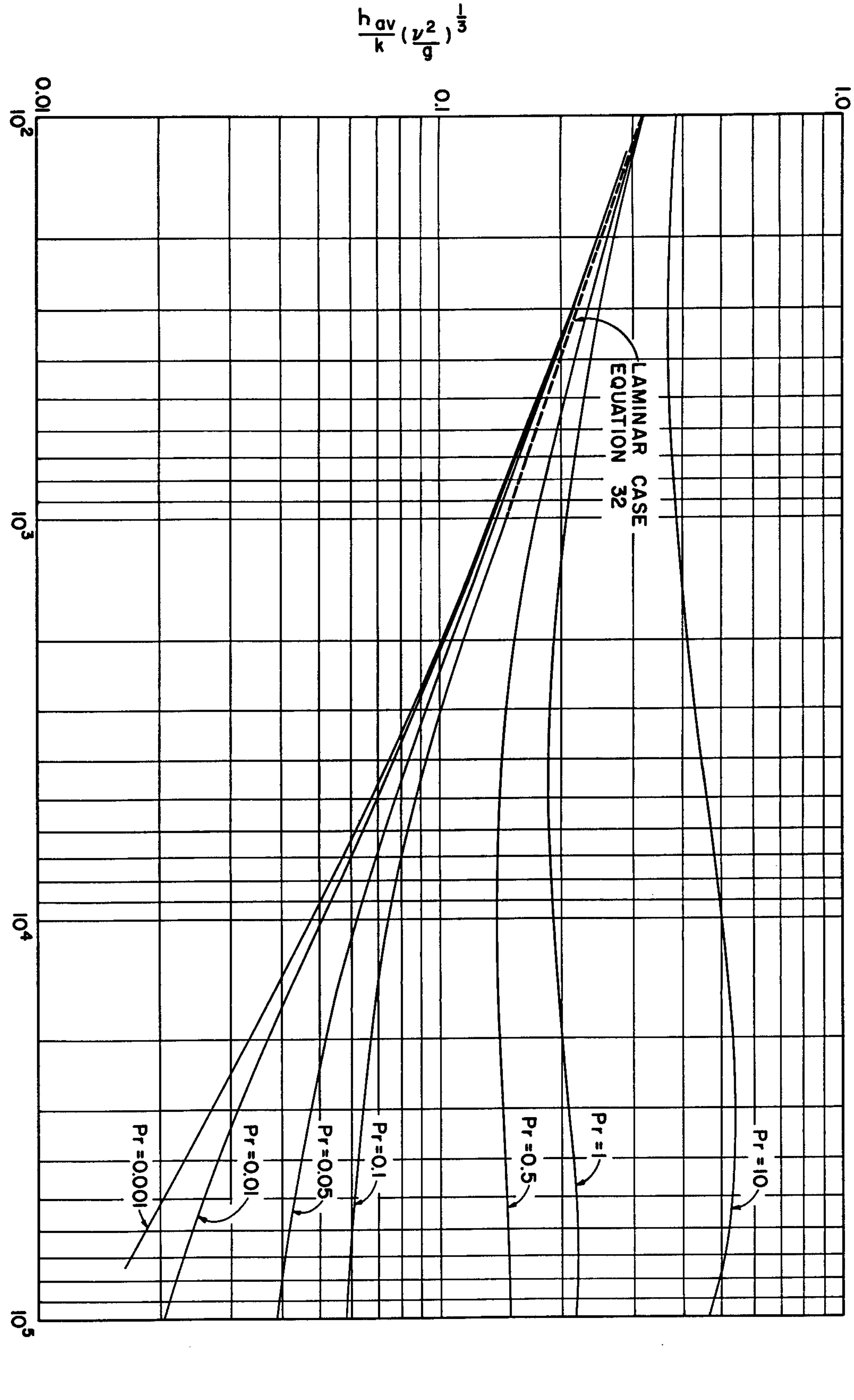

FIGURE 6 - AVERAGE HEAT TRANSFER COEFFICIENT VS FLOW RATE

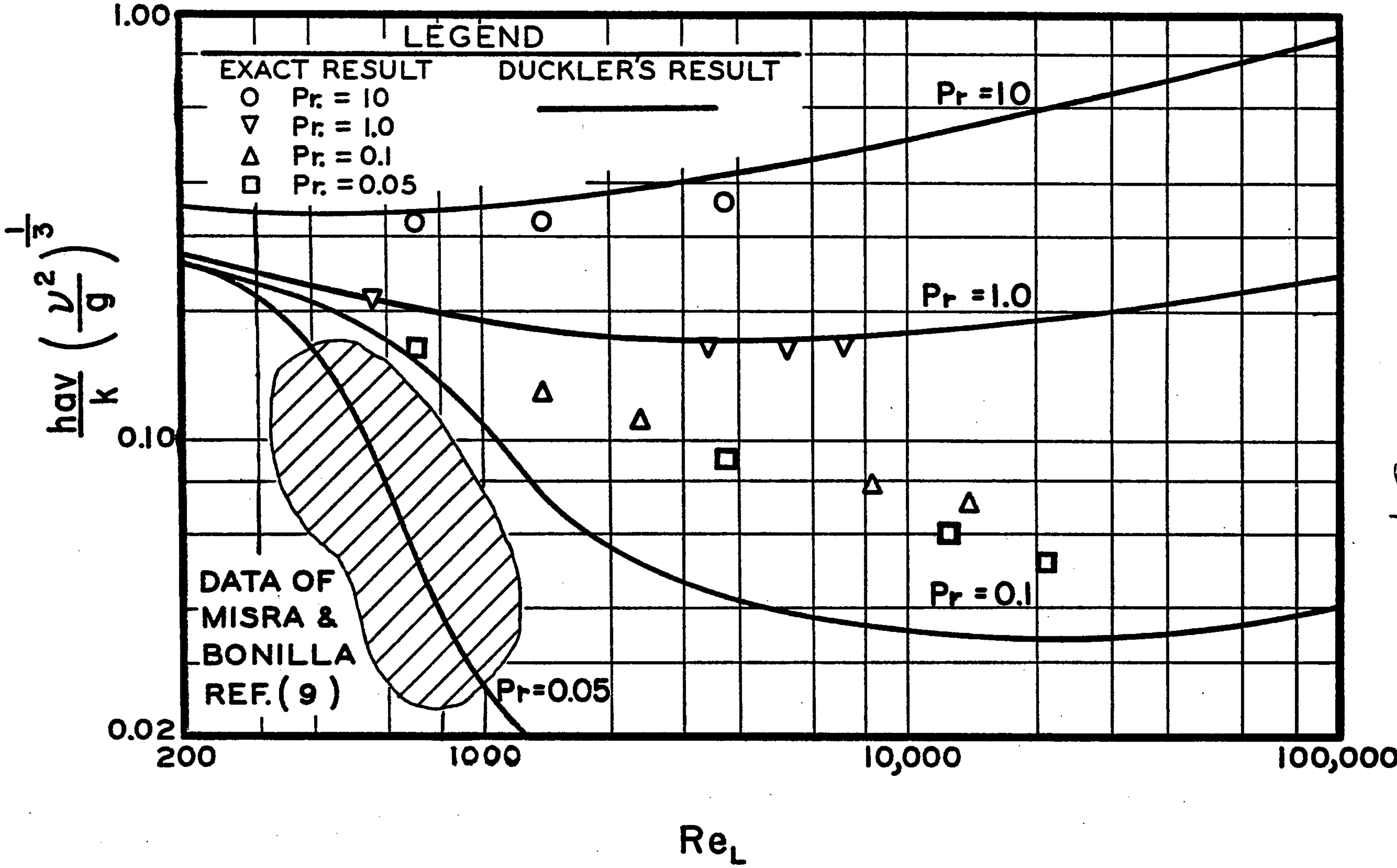

FIG. 7 COMPARISON OF DUCKLER'S WITH EXACT RESULTS

(TAKEN FROM FIGURES A 11 AND 15 OF REFERENCE 3)